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# SIMULATION OF THE PURCELL EFFECT BY CIRCUIT THEORY METHODS

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**МОДЕЛЮВАННЯ ЕФЕКТУ ПАРСЕЛЛА МЕТОДАМИ ТЕОРІЇ КІЛ**

У доповіді розглянуто так званий ефект Парселла, який стимулює швидкий розряд високодобротної структури з більшим часом життя ("квантова точка") при взаємодії його з низькодобротним резонатором. З погляду класичної теорії кіл використовуються узгоджувальні властивості мостових структур.

The report examines the so-called Purcell effect, which stimulates the rapid discharge of a high-Q structure with a long lifetime ("quantum dot") when it interacts with a low-Q resonator. From the point of view of classical circuit theory, the matching properties of lattice structures are used.

**Keywords:** PURCELL EFFECT, MICROWAVE RESONATOR, LATTICE 4-POLE, Q-FACTOR, GROUP DELAY.

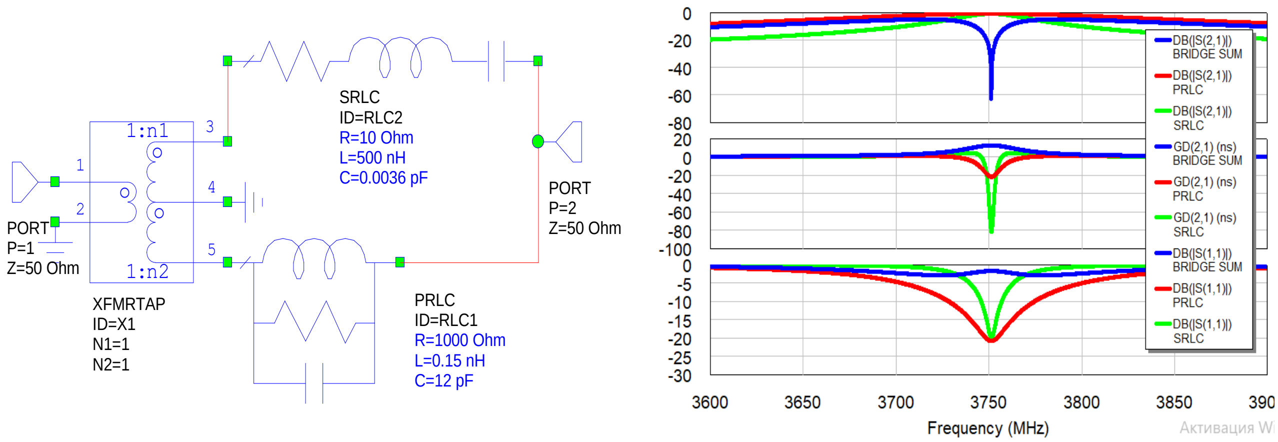
Purcell's famous paper [1] on the change in the probability of spontaneously emitting a nuclear magnetic moment associated with a resonant electric circuit was published in 1946, and in 1948 the book "Principles of Microwave Circuits" appeared [2], one of the co-authors of which was E. M. Purcell. It seems very interesting to analyze the effect described in [1] with the help of methods of analysis using microwave circuits, bridge circuits of quadrupoles, and Smith diagrams, which, as follows from [2], the future Nobel laureate possessed perfectly.

Modeling of the Purcell effect and similar quantum resonance effects such as Fano resonance and Electromagnetically Induced Transparency (EIT) using metamaterial cells has been described quite comprehensively in the modern scientific literature [3-5]. It was demonstrated in [6] that the EIT phenomenon is characterized by a "change" of the sign of the group delay time (GD) concerning the GD of individual resonators. Let us consider a bridge (lattice) quadrupole formed by parallel and series resonant circuits with different unloaded quality factors of fit (Fig. 1 (a)). According to [7], they are defined as:

$$Q_{series} = \omega_0 \cdot \frac{L}{R_s} = \frac{1}{\omega_0} \cdot R_s \cdot C \quad (1)$$

$$Q_{parallel} = \frac{R_p}{\omega_0} \cdot L = \omega_0 \cdot R_p \cdot C \quad (2)$$

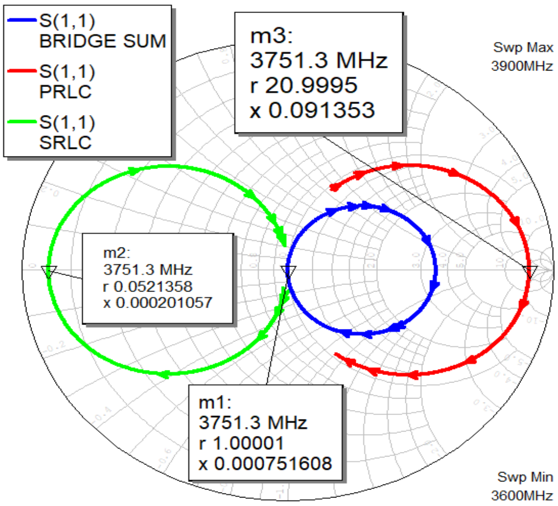
Fig.(1(b)) shows the scattering matrix parameters of the considered 4-pole. The bottom plot is the transmission coefficients S21, the middle plot is the delay times – GD (S12), and the upper plot is the reflection coefficients S11. The red curves refer to the low-Q parallel oscillating circuit, the green curves refer to the good-Q oscillating circuit, and the blue curves refer to the characteristics of the entire 4-pole. As we see, as well as in the realization of the Purcell effect, there is a noticeable decrease in the "discharge time" (GD in our case), as well as the "inversion" of the GD sign.




a) b)

Fig. 1. A) – equivalent substitution scheme of the investigated microwave structure, b) parameters of the scattering matrices of the investigated structures: bottom graph – transmission coefficients S21, middle – GD (S12), upper – reflection coefficients S11.

Let us note that in the "fast discharge" mode at the resonant frequency the reflection coefficient of the whole quadrupole is practically zero (blue curve on the upper graph), i.e. the investigated 4-pole is almost perfectly matched. In the second half of the 40s of the 20$^{th}$ century, when the Purcell effect was discovered and described, the matching of microwave devices was performed with the help of the so-called Smith diagrams, which, as noted, are also used for this purpose in [2]. Fig. 2 shows the Smith diagram for the reflection coefficients (S11) of the parallel (red curve) and series (green curve) resonators, as well as the S11 characteristic of the whole quadrupole (blue curve). Markers on them mark the values of normalized input impedances. The analysis of the presented curves shows that separately taken resonators are uncoordinated and therefore most of the energy "reflect" in the direction of the "source", and in the mode of "joint" radiation – perfectly coordinated and practically do not reflect anything in the direction of the "source".




The mentioned reason – mutual coordination of resonators in case of their use in the mode of "parallel channels" of radiation, is both the reason of the Purcell effect and, partly, of EIT. This can be caused by the so-called "locked mode" of the 4-pole bridge [8].

Fig. 2: Smith diagrams of the investigated 4-pole; reflection coefficients (S11): red curve - parallel resonator, green curve - series resonator, blue curve - total 4-pole.

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