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# ALIGNMENT JITTER EVALUATION IN DIGITAL TRANSMISSION SYSTEMS USING QUEUING MODEL

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## ОЦІНЮВАННЯ ДЖИТЕРУ УЗГОДЖЕННЯ В ЦИФРОВИХ СИСТЕМАХ ПЕРЕДАЧІ З ВИКОРИСТАННЯМ МОДЕЛІ ЧЕРГ

В роботі розглядається можливість застосування математичного апарату теорії масового обслуговування (модель черг на обслуговування) з метою оцінювання параметрів узгодження швидкостей сигналів в цифрових системах передавання, зокрема, джитеру часу очікування.

The idea of applying queuing model known from the teletraffic theory to evaluate the signals alignment parameters (in particular, waiting time jitter) in the digital transmission systems has been considered.

Any interaction of digital signals (DS) is based on the data exchange (writing and reading) e.g. multiplexing/demultiplexing in PDH/SDH systems, switching centers (SC) connections, ATM cells and IP packets processing etc. It leads to the inevitable data losses (slips[1] of bits, bytes, frames, packets etc.) and requires some form of digital signals alignment [1, 2]. The alignment methods include bit/byte justification (stuffing), pointers (in SDH), controlled slips between SC, limitation of packet length. But only digital signals justification during multiplexing excludes slips as opposed to controlled frame slips between SC or packet loss.

There are well-known traditional approaches to evaluate the digital alignment effectiveness such as slip estimation in digital switching/transmission systems or PDV (packet delay variation) concept in packet networks [1, 2]. On the other hand, it may be useful to analyze the same processes from the other point of view – for example, by applying queuing model known from the classical teletraffic theory. In particular, we can consider alignment-related parameters such as

- PDV for packets and ATM cells,
- TIE accumulation in ISC[2] chain interconnection,
- the mean Time Error (TE) caused by a latency jitter in PDH/SDH systems

as event flows in a queue for service.

To illustrate the approach, let us consider TE event flow under the assumption of Poisson (simplest) flow [3, 4]. Such a flow should meet the following general conditions:

- The flow is uniform, homogeneous; all events are equal.

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[1] Slip is the loss of data or the appearance of extra positions in a digital stream.
[2] ISC – International Switching Center

Only the event arrivals (kick-off moments) are considered.

– No aftereffect (no or limited correlation between events on the different non-overlapping time intervals).
– Stationary time series/flow.

In general case, the stationarity condition is not met strictly for TE event flow because TE depends on time in the presence of constant frequency offset. But on the relatively short time intervals (corresponding to queue length) this dependence can be neglected and therefore TE flow can be considered locally stationary.

Let us consider as an example the conventional PDH multiplexing system $4 \times$ E12 = 1 E22 ($4 \times 2048$ kbit/s = 8448 kbit/s)[1]. The main parameters of writing/reading processes in the multiplexer elastic memory (EM) are:

- nominal E22 frame duration $T_{\text{frame}}$;
- nominal E22 frame frequency $f_{\text{FRnom}}$;
- nominal writing frequency of E12 tributary signal $f_{\text{Wnom}}$;
- nominal reading frequency of E12 tributary $f_{\text{Rnom}}$.

One staffing bit is allocated for matching every E12 tributary signal in each E22 frame, i.e. maximal staffing frequency is equal to the nominal E22 frame frequency: $f_{\text{st max}} = f_{\text{FRnom}}$.

The nominal justification rate is defined as:

$$\rho_{\text{nom}} = (f_{\text{Rnom}} - f_{\text{Wnom}}) / f_{\text{st max}} = 0.4242$$

The real justification rate $\rho$ can change over the interval $0 < \rho < 1$, so the bit staffing will occur irregularly. The real justification rate $\rho$ can be considered as fractional frequency offset $y$ of the writing/reading frequency difference $\Delta f_{\text{WR}} = (f_{\text{R}} - f_{\text{W}})$ relative the staffing rate $f_{\text{st max}}$.

The reciprocal of the justification rate $1/\rho$ shows how many frames are passed for one staffing on average, e.g. $1/\rho = 2.37$ signifies one staffing operation during two-three frames. The staffing irregularity causes additional waiting time jitter (or latency jitter) in the multiplexed signal [5].

It is interesting to evaluate the digital alignment processes and, in particular, waiting time jitter, using queuing theory. Any queuing system is defined by the following main parameters [4]:

- $\lambda$ – arrival rate (request rate);
- $\mu$ – service rate;
- $\rho = \lambda/\mu$ – utilization factor of the server (e.g. elastic memory (EM), controller, etc.);
- $t_s$ – service time;
- mean service time: $E(t_s) = 1/\mu$;

Stability condition of a queuing system is defined as $\lambda < \mu$ or $\lambda/\mu < 1$. Therefore, the system state can be described by the probability P:

$\rho = \lambda/\mu \equiv$ P [system is busy], and

$1 - \rho \equiv$ P [system is idle].

[1]The multiplexing principle and frame structure are defined in ITU-T Recommendation G.742.

In the case of waiting time jitter in the above-mentioned PDH multiplexing system $4 \times E12 = E22$, the queuing system parameters to be analyzed are:the mean waiting time of staffing;

- the mean time on the general justification time (mean waiting time + mean service time ) and
- the waiting time deviation.

*Request rate* can be defined as

$$\lambda \equiv \Delta f_{WR} = (f_{Rnom} - f_{Wnom}),$$

where $f_{Rnom}$ and $f_{Wnom}$ – nominal writing and reading frequencies in the EM.

*Service rate* is $\mu \equiv f_{st\,max} = f_{FRnom.}$

*Utilization factor* of the PDH equipment EM:

$$\rho \equiv \lambda/\mu \equiv y \text{ (fractional frequency offset)}$$

We analyze TE event flow using *M/D/*1 queuing system model[1] that means [4]:

- Poisson (simplest) arrival process;
- Deterministic service time;
- One server.

The calculations using this queuing model for the above-mentioned conditions gave the following waiting time jitter mathematical expectation (average value) E(tw) and deviation σ(tw):

$E(tw) = 36.9\ \mu s \approx 37\ \%\ T_{frame}$ ;  $\sigma(tw) = 61.9\ \mu s \approx 62\ \%\ T_{frame}$

To compare the obtained results with traditional "common sense" approach, the modelling of TIE during bit staffing process in PDH multiplexer was performed for the different values of the phase offset threshold (staffing request). The indirect estimation of the average waiting time jitter for $\rho_{nom} \equiv y$ is 36 μs that is close to queueing model M/D/1 results. The modelling has shown that the average latency (waiting time jitter) essentially depends on the used threshold level and is between [26.7 μs, 42.6 μs].

The idea of using queuing models for digital transmission system parameters evaluation may be useful in both conventional and state-of-the art telecom equipment. The presented approach can be further developed and applied for the analysis and optimization of the various digital signals alignment processes.

**References**


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[1] The queue has an infinite storage capacity.